Loop Quantum Gravity

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- Elements of Loop Quantum Gravity (LQG)
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- Which new physics takes over near singularities (black holes, big bang)?
- What is the origin of dark energy?
- How does the UV completion of effective theories (e.g. perturbative QG) look like?

For more than 70 years physicists are looking for a unified theory of general relativity and quantum mechanics – so far w/o success.

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Why?
Gravity = Geometry, Curvature = Matter Energy Density

Einstein’s equations

\[ R_{\mu\nu}[g] - \frac{1}{2} R[g] \, g_{\mu\nu} = 8\pi G \, T_{\mu\nu}[g] \]

Background independence: Geometry g not prescribed but dynamically determined by matter energy density T.
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Background independence and backreaction (gravitational waves)
QFT on CST rigorously developed [Wald 90’s, Hollands & Wald 00’s, Fredenhagen, Brunetti, Verch 00’s]

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But: fundamentally background dep. Haag – Kastler formulation:
If $\mathcal{O}, \mathcal{O}'$ spacelike separated wrt $g_0$ then $[\mathcal{A}(\mathcal{O}), \mathcal{A}(\mathcal{O}')] = 0$
Ordinary QFT: photons propagate on rigid spacetime
BI QFT: Fuzzy (fluctuating) lightcone
The structure crucial for ordinary QFT

\[ g_0 \Rightarrow (x - y)^2 < 0 \Rightarrow A \]
Background Light Cone Algebra

collapses when \( g_0 \) not available.

ignores gravitational backreaction.

invalid approx. in extreme cosm. & astrophys. situat.

Perturbative approach

\[ g = g_0 + h \]
Total Metric Background Perturbation (Graviton)

violates BI, inacceptable due to non-renormalisability, merely effect. graviton QFT over \( g_0 \).
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Quantum - Einstein - Equations

- Relaxation of principles of QFT on CST.
- QFT on diff. mfd. M rather than QFT on spacetime \((M, g_0)\).
- Theory that works even if notion of classical (smooth) metric breaks down ....
- but contains QFT on CST for all backgrounds.
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The Challenge of Quantum Gravity
Elements of Loop Quantum Gravity (LQG)
Research Directions

Principle of Background Independence
Disclaimer

LQG is/does not

- a unified theory of all interactions.
- predict the matter content and dimension of the world.

Instead, LQG is/ tries

- a particular incarnation of the canonical approach to QFT including GR.
- to consistently combine the salient principles of QFT and GR.
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Canonical Formulation: $M \cong \mathbb{R} \times \sigma$ globally hyperbolic
Due to spinorial matter, forced to use connections.

- Canonical variables: \( (A^j_a, E^a_j) \) [Ashtekar, Barbero, Immirzi, Sen 80's]
- Poisson brackets: \( \{A^j_a(x), E^b_k(y)\} = G \delta^b_a \delta^j_k \delta(x, y) \)
- Constraint = Gauss Law: \( C_j = D_a E^a_j = 0 \)
- Phase space of YM sector of weak interaction!
- However, different “dynamics”:

\[
H_{\text{can}}[L, S] = \int_\sigma d^3 x \left\{ \frac{L \text{Tr}(F_{ab} E^a E^b) + \text{Tr}(F_{ab} E^a) \text{Tr}(SE^b)}{|\det(E)|^{1/2}} \right. \\
\left. + L \wedge |\det(E)|^{1/2} + \text{more} \right\} + \text{matter terms}
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- EOM+Gaus Constraint+\( H_{\text{can}}[L, S] = 0 \ \forall \ L, S \leftrightarrow \text{Einstein-Equations} \)
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Task:

- **Step 1**: Find repr. of CCR s.t. “Hamiltonian” becomes d.d. op.
- **Step 2**: Solve quantum constraints $H_{\text{can}}(L, S)\Psi = 0$
- **Step 3**: Equip solution space with physical inner product

Even step 1 is a **nightmare**: how to tame the monster?

Idea: regulate by smearing the fields “economically” s.t. well defined op. remains when removing regulator.

- Cosmological term $\Rightarrow$ $E$ must be smeared in precisely 2D
- Einstein term $\Rightarrow$ $A$ must be smeared in precisely 1D

Works **universally** for all terms and matter couplings!

Reason: diffeomorphism invariance

**Dynamics** forces us to consider the Holonomy-Flux Algebra
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Holonomy-Flux algebra [Rovelli& Smolin, Gambini & Pullin 80’s]

- Magnet. dof.: Holonomy (Wilson-Loop; lattice gauge th. inspired)

\[
A(e) = \mathcal{P} \exp\left(\int_e A\right)
\]

- Electr. dof: flux

\[
E_f(S) = \int_S \epsilon_{abc} \langle E^a, f \rangle \, dx^b \wedge dx^c
\]

- Poisson – brackets:

\[
\{E_f(S), A(e)\} = G A(e_1) f(S \cap e) A(e_2); \quad e = e_1 \circ e_2, \quad e_1 \cap e_2 = e \cap S
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Disclaimer
Classical Formulation
Quantum Theory

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- Reality conditions:
  \[ \overline{A(e)} = [A(e^{-1})]^T, \quad E_f(S) = E_f(S) \]

- Defines abstract Poisson*–algebra \( \mathcal{A} \)

- Bundle automorphisms \( \mathcal{G} \cong \mathcal{G} \times \text{Diff}(\sigma) \) act by Poisson automorphisms on \( \mathcal{A} \) e.g.
  \[ \alpha_g = \exp(\{\int \lambda^j C_j, .\}), \quad g = \exp(\lambda^j \tau_j) \]
  \[ \alpha_g(A(e)) = g(b(e)) A(e) g(f(e))^{-1}, \quad \alpha_\varphi(A(e)) = A(\varphi(e)) \]
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Holonomy-Flux algebra [Rovelli& Smolin, Gambini & Pullin 80’s]

- Magnet. dof.: Holonomy (Wilson-Loop; lattice gauge th. inspired)
  \[ A(e) = \mathcal{P} \exp(\int_e A) \]

- Electr. dof: flux
  \[ E_f(S) = \int_S \epsilon_{abc} < E^a, f > dx^b \wedge dx^c \]

- Poisson – brackets:
  \[ \{E_f(S), A(e)\} = G A(e_1) f(S \cap e) A(e_2); \quad e = e_1 \circ e_2, \quad e_1 \cap e_2 = e \cap S \]

- Reality conditions:
  \[ \overline{A(e)} = [A(e^{-1})]^T, \quad \overline{E_f(S)} = E_f(S) \]

- Defines abstract Poisson*–algebra \( \mathfrak{A} \)

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Representation Theory

- In QFT no Stone – von Neumann uniqueness Theorem!!!
- Idea: Use symmetry principles to select a representation (cf. Poincaré symmetry for QFT in Minkowski space)
- Here $\mathcal{G}$ covariance of $H_{\text{can}}(L, S)$

$$\alpha_{g, \varphi}[H_{\text{can}}(L, S)] = H_{\text{can}}([\varphi^{-1}]^* L, [\varphi^{-1}]^* S)$$

- Cyclic representations are classified by analysing corresponding states (GNS theorem)

**Theorem** [Ashtekar, Isham, Lewandowski 92-93], [Sahlmann 02], [L., Okolow, S., T.T. 03-05], [Fleischhack 04]

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wave functions of $\mathcal{H}_{\text{phys}}$

$$\psi(A) = \psi_\gamma(A(e_1),..,A(e_N)), \quad \psi_\gamma : \text{SU}(2)^N \rightarrow \mathbb{C}$$

- Holonomy = multiplication – operator

$$[\hat{A}(e) \psi](A) := A(e) \psi(A)$$

- Flux = derivative – operator

$$[\hat{E}_j(S) \psi](A) := i\hbar \{E_j(S), \psi(A)\}$$

- Scalar product

$$<\psi,\psi'> := \int_{\text{SU}(2)^N} d\mu_H(h_1) .. d\mu_H(h_N) \bar{\psi}_\gamma(h_1,..,h_N) \psi'_{\gamma}(h_1,..,h_N)$$

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Spin Network Basis $T_{\gamma,j,l} \sim H_j$ Hermite Polynomials
Colour Coding of Spin Quantum Numbers
Simplex and Dual Graph
Dual Diamond: Coloured, simplicial cell complex (Triangulation)

Animation:
http://www.einstein-online.info/de/vertiefung/Spinnetzwerke/index.html.
Does this representation support the Hamiltonian?

- Thm: Let $\mathcal{H}_\gamma =$ closed lin. span of SNWF over $\gamma$
  
  \[ \Rightarrow \mathcal{H} = \bigoplus_\gamma \mathcal{H}_\gamma \]

- Not lattice gauge theory on fixed graph
- But lattice gauge theory on all graphs
- Continuum limit already taken
- Thm2: $H_{\text{can}}$ densely defined on $\mathcal{H}$
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Comparison with YM theory on cubic lattice

- Yang – Mills on \((R^4, \eta)\) [Kogut & Susskind 74]

\[
H_{YM,\gamma} = \frac{\hbar}{2 g^2} \epsilon \sum_{v \in V(\gamma)} \sum_{e \cap e' \cap e'' = v} \text{Tr} \left( E(S_{e''})^2 + [2 - A(\alpha_{vee'}) - A(\alpha^a_{vee'})^{-1}] \right)
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- Gravity on \(\mathbb{R} \times \sigma\) [T.T. 90’s]

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V_v = \sqrt{\left| \sum_{e \cap e' \cap e'' = v} \sigma(e, e', e'') \text{Tr} (E(S_e) E(S_{e'}) E(S_{e''})) \right|}
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  - Canonical:
    - Improved Constraint Algebra, Consistent Discretisations [Gambini & Pullin 00’s], Master Constraint Programme, Deparametrisation, Semiclassical Limit.
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\[
\{H_{\text{can}}(L, 0), H_{\text{can}}(L', 0)\} = -H_{\text{can}}(0, \frac{E}{|\det(E)|^{1/2}} \cdot [L \, dL' - L' \, dL])
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- Question: If \( U(\varphi)\Psi = \Psi \), is it true that

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\langle \Psi, [H_{\text{can}}(L, 0), H_{\text{can}}(L', 0)]\psi \rangle = 0
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- Answer: yes, but can we close algebra off shell and thereby fix q’ion ambiguities?

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Idea: Extend $\mathcal{H}$ to certain distributions [Gambini, Marolf, Lewandowski, Pullin 90’s] on which $H_{\text{can}}(0, S)$ exists.

Already promising results for the $U(1)^3$ model ($G \rightarrow 0$ limit)
Improved Constraint Algebra [Henderson, Laddha, Tomlin, Varadarajan 10’s]

Classically

\[
\{H_{\text{can}}(L, 0), H_{\text{can}}(L', 0)\} = -H_{\text{can}}(0, \frac{E}{|\det(E)|^{1/2}} \cdot [L\, dL' - L'\, dL])
\]

Question: If \(U(\varphi)\psi = \psi\), is it true that

\[
<\psi, [H_{\text{can}}(L, 0), H_{\text{can}}(L', 0)]\psi >= 0
\]

Answer: yes, but can we close algebra off shell and thereby fix q’ion ambiguities?

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- SFM must define a rigging map, heuristically [Reisenberger, Rovelli 90’s]

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\eta[\psi] := \langle \psi, \delta[H_{\text{can}}] \rangle := \int d\mu(L, S) \langle \psi, \exp(iH_{\text{can}}(S, L)) \rangle.
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- Integrate out \(\lambda\) and introduce funct. derivatives

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- SFM f. quantum groups and cosmological const. [Han; Fairbairn, Meusburger 10]
Many promising results:

- Sum over $\kappa$ controllable by GFT formulation (Feynman graph expansion) [Baratin, Bonzom, Freidel, Gurau, Krajewski, Oriti, Rivasseau, ...00’s] SFM

\[
S_{\text{GFT}}[\phi] = \int_{G^n} d\mu_H(g) \left[ \phi(g)^2 + V(\phi(g)) \right]
\]

- Appropriate framework for studying renormalisation and perfect discretisations of spacetime diffeomorphisms [Bahr, Dittrich 10’s]

- Graviton propagator [Alesci, Bianchi, Magliaro, Perini, Rovelli, Speziale 00’s]

- Semiclassical limit (large spin) related to Regge Calculus [Bahr, Barrett, Conrady, Dittrich, Freidel, Hellmann, Speziale 10’s]

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- Cosmology [Viadotto, Rovelli] and BH entropy [Bianchi, Rovelli] from SFM
Hawking’s Area Theorem (classical GR) and Hawking’s radiation (QFT in classical ST) strongly suggest that BH have an entropy

\[ S_{BH} = \frac{\text{Ar}(H \cap \sigma)}{4L_P^2} \]

What is the microscopic origin of this entropy?
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It from Bit? ['t Hooft, Susskind, Wheeler]
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- What is the microscopic origin of this entropy?
- Idea: Count number \( N \) of eigenstates of operator \( \text{Ar}(H \cap \sigma) \), define \( S \equiv \ln(N) \) [Krasnov, Rovelli 90’s]
- Semiclassical quantum boundary conditions make sure that:
  - not any surface but isolated horizon (equilibrium)
  - \( \text{Ar}(H \cap \sigma) \) is gauge invariant (observable)
  - \( S \) correctly accounted for by bdry CS dof
- Upon fixing a free parameter to a universal (matter independent) value [Ashtekar, Baez, Corichi, Krasnov 00’s]

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Warning: All spins contribute, no bit picture!
Many generalisations:

- **Improved Counting methods** [Barbero, Agullo, Borja, Diaz-Polo, Sahlmann, Villasenor, ...]
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